Sample Examination V

Section I Part A

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

1. If $y = (2x^2 + 1)^4$, then $\frac{dy}{dx} =$

(A) $16x^3$

(B) $4(2x^2 + 1)^3$

(C) $4x(2x^2 + 1)^3$

(D) $16(2x^2 + 1)^3$

(E) $16x(2x^2 + 1)^3$

Answer

2. $\int x\sqrt{x^2+1}\,dx =$

(A) $\frac{x}{\sqrt{x^2+1}} + C$

(B) $\frac{3}{4}(x^2+1)^{\frac{3}{2}} + C$

(C) $\frac{1}{3}(x^2+1)^{\frac{3}{2}} + C$

(D) $\frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$

(E) $\frac{1}{3}x^2(x^2+1)^{\frac{3}{2}} + C$

Answer

3. If $f(x) = x\sqrt[3]{x}$, then $f'(x) =$

(A) $4x^3$ (B) $\frac{3}{7}x^{\frac{7}{3}}$ (C) $\frac{4}{3}x^{\frac{1}{3}}$ (D) $\frac{1}{3}x^{\frac{1}{3}}$ (E) $\frac{1}{3}x^{-\frac{2}{3}}$

Answer

4. A curve in the plane is defined parametrically by the equations $x = 2t + 3$ and $y = t^2 + 2t$. An equation of the line tangent to the curve at $t = 1$ is

(A) $y = 2x - 7$

(B) $y = x - 2$

(C) $y = 2x$

(D) $y = 2x - 1$

(E) $y = \frac{1}{2}x + \frac{1}{2}$

Answer

5. $\int_0^8 \frac{1}{\sqrt[3]{8-x}}\,dx$ is

(A) -6 (B) 2 (C) 6 (D) 12 (E) nonexistent

Answer

6. $\int x \sin x\,dx =$

(A) $-\frac{1}{2}x^2 \cos x + C$

(B) $-x \cos x + C$

(C) $x \cos x - \sin x + C$

(D) $-x \cos x + \sin x + C$

(E) $-x \cos x - \sin x + C$

Answer

7. Let f be a differentiable function for all x. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)\,dx = f(x)$

II. $\int_3^x f'(x)\,dx = f(x)$

III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

Answer

8. If $\sin(xy) = x^2$, then $\frac{dy}{dx} =$

(A) $2x\sec(xy)$

(B) $\frac{\sec(xy)}{x^2}$

(C) $2x\sec(xy) - y$

(D) $\frac{2x\sec(xy)}{y}$

(E) $\frac{2x\sec(xy) - y}{x}$

Answer

9. For all x in the closed interval $[1,4]$, the function g is concave upwards. Which of the following tables could be the values of $g(x)$?

(A)

x	$g(x)$
1	-10
2	-7
3	-6
4	-2

(B)

x	$g(x)$
1	4
2	6
3	9
4	14

(C)

x	$g(x)$
1	0
2	5
3	7
4	12

(D)

x	$g(x)$
1	-4
2	-6
3	-8
4	-10

(E)

x	$g(x)$
1	-2
2	-1
3	5
4	3

Answer

10. $\int \frac{dx}{x^2+4x} =$

(A) $\int \frac{dx}{x} + \int \frac{dx}{x+4}$

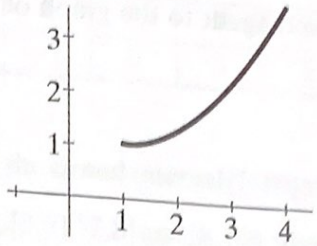
(B) $\int \frac{dx}{x^2} + \int \frac{dx}{4x}$

(C) $\int \frac{dx}{x} - \int \frac{dx}{x+4}$

(D) $\int \frac{dx}{4x} + \int \frac{dx}{4(x+4)}$

(E) $\int \frac{dx}{4x} - \int \frac{dx}{4(x+4)}$

Answer



Graph of f

11. The graph of a function f is shown above for $1 \le x \le 4$. Of the following, which has the least value?

(A) $\int_1^4 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^4 f(x)\,dx$ with 3 subdivisions of equal length

(C) Right Riemann sum approximation of $\int_1^4 f(x)\,dx$ with 3 subdivisions of equal length

(D) Midpoint Riemann sum approximation of $\int_1^4 f(x)\,dx$ with 3 subdivisions of equal length

(E) Trapezoidal sum approximation of $\int_1^4 f(x)\,dx$ with 3 subdivisions of equal length

Answer

12. What is the 20th derivative of $y = \sin(2x)$?

(A) $-2^{20}\sin(2x)$

(B) $2^{20}\sin(2x)$

(C) $-2^{19}\cos(2x)$

(D) $2^{20}\cos(2x)$

(E) $2^{21}\cos(2x)$

Answer

13. What is an equation of the line tangent to the graph of $f(x) = 7x - x^2$ at the point where $f'(x) = 3$?

(A) $y = 5x - 10$

(B) $y = 3x + 4$

(C) $y = 3x + 8$

(D) $y = 3x - 10$

(E) $y = 3x - 16$

Answer

14. Suppose that $f(x)$ is a twice-differentiable function on the closed interval $[a,b]$. If there is a number $c, a < c < b$, for which $f'(c) = 0$, which of the following must be true?

I. $f(a) = f(b)$

II. f has a relative extremum at $x = c$.

III. f has a point of inflection at $x = c$.

(A) None

(B) I only

(C) II only

(D) I and II

(E) II and III

Answer

x	1	3	7	8	10
$f(x)$	−2	0	3	3	−4

15. The function f is continuous on the closed interval $[1,10]$ and has the values shown in the table above. Using the intervals $[1,3]$, $[3,7]$, $[7,8]$ and $[8,10]$, what is the approximation of $\int_1^{10} f(x)\,dx$ obtained from a right Riemann sum?

(A) 5 (B) 6 (C) 7 (D) 13 (E) 23

Answer

16. What are the first four nonzero terms in the power series expansion of e^{-4x} about $x = 0$?

(A) $1 + x + \frac{x^2}{2} + \frac{x^3}{3}$

(B) $1 - 4x + 8x^2 - 32x^3$

(C) $1 - 4x - 2x^2 - \frac{2}{3}x^3$

(D) $1 - 4x + 8x^2 - \frac{32}{3}x^3$

(E) $1 - 4x + 8x^2 - \frac{64}{3}x^3$

Answer

17. On the open interval $a < x < b$ the graph of g', the derivative of g, is continuous and differentiable. On this interval the graph of g' has several local maximums, several local minimums and several zeros. The graph of g changes from concave down to concave up at which points on the graph of g' ?

(A) At the zeros where g' changes from positive to negative

(B) At the zeros where g' changes from negative to positive

(C) At the local maximums of g'

(D) At the local minimums of g'

(E) At the zeros of g'

Answer

18. $\int_e^{e^2} \frac{dx}{x \ln x} =$

(A) $\ln 2$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) e

Answer

19. Let $f(x)$ be a differentiable function on the interval $0 \le x \le 1$, and let $g(x) = f(3x)$. The table below gives values of $f'(x)$, the derivative of $f(x)$. What is the value of $g'(0.1)$?

x	0.1	0.2	0.3	0.4	0.5	0.6
$f'(x)$	1.01	1.041	1.096	1.179	1.298	1.486

(A) 1.010 (B) 1.096 (C) 1.486 (D) 3.030 (E) 3.288

Answer

20. Which of the following integrals gives the total area of the region shared by both polar curves $r = 2\cos\theta$ and $r = 2\sin\theta$?

(A) $2\int_0^{\frac{\pi}{4}} \sin^2\theta\, d\theta$

(B) $4\int_0^{\frac{\pi}{4}} \sin^2\theta\, d\theta$

(C) $2\int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta$

(D) $4\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$

(E) $2\int_0^{\frac{\pi}{4}} (\cos^2\theta - \sin^2\theta)\, d\theta$

Answer

21. $\lim_{h \to 0} \dfrac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h}$ is

(A) 0

(B) $10x^3 - 15x$

(C) $10x^4 + 15x^2$

(D) $10x^4 - 15x^2$

(E) $-10x^4 + 15x^2$

Answer

22. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)\,dx = 6$, then $\int_8^4 f(x)\,dx =$

(A) -16 (B) -6 (C) -4 (D) 4 (E) 16

Answer

23. If the graph of $y = x^3 + ax^2 + bx - 8$ has a point of inflection at $(2,0)$, what is the value of b?

(A) 0

(B) 4

(C) 8

(D) 12

(E) The value of b cannot be determined from the given information.

Answer

24. The position of a particle in the xy-plane is given by $x = 4t^2$ and $y = \sqrt{t}$. At $t = 4$, the acceleration vector is

(A) $\left\langle 8, -\frac{1}{64} \right\rangle$

(B) $\left\langle 8, -\frac{1}{32} \right\rangle$

(C) $\left\langle 8, \frac{1}{32} \right\rangle$

(D) $\left\langle 32, -\frac{1}{32} \right\rangle$

(E) $\left\langle 32, \frac{1}{4} \right\rangle$

Answer

25. If f is a continuous function on the closed interval $[a,b]$, which of the following statements are NOT necessarily true?

I. f has a minimum on $[a,b]$.

II. f has a maximum on $[a,b]$.

III. $f'(c) = 0$ for some number $c, a < c < b$.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

Answer

26. What are all the values of x for which the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1}\frac{x^n}{n} + \cdots$ converges?

(A) $-1 \le x \le 1$

(B) $-1 \le x < 1$

(C) $-1 < x \le 1$

(D) $-1 < x < 1$

(E) All real numbers x

Answer

27. $\sum_{n=0}^{\infty} \frac{(-1)^n (\pi)^{2n}}{(2n)!} =$

(A) 1 (B) -1 (C) π (D) $\frac{\pi}{2}$ (E) e^{π}

Answer

28. If $\frac{dy}{dx} = \frac{x}{y}$ and $y(3) = 4$, then

(A) $x^2 - y^2 = -7$

(B) $x^2 + y^2 = 5^2$

(C) $x^2 - y^2 = 7$

(D) $y^2 - x^2 = 5$

(E) $y^2 + x^2 = 7^2$

Answer

Section I Part B

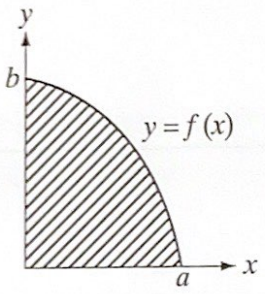
Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may be used on this part of the exam.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)



29. Let $f(x)$ be a continuous function and let A be the area of the shaded region in the figure above. Which of the following must be true?

I. $A = \int_0^a f(x)\,dx$ II. $A = \int_0^b f^{-1}(x)\,dx$ III. $A = \int_0^b f^{-1}(y)\,dy$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

Answer

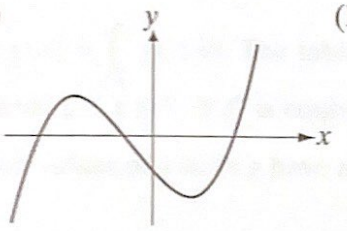
30. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{x^n}{2n}$. What is the value of $f^{(4)}(0)$, the fourth derivative of f at $x = 0$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Answer

31. If $f'(x) = (x - a)(x - b)(x - c)$ and $a < b < c$, then which of the following could be the graph of $f(x)$?

(A)



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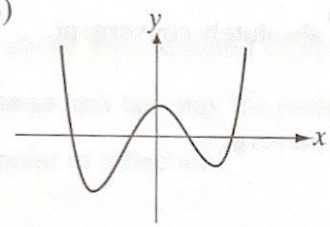

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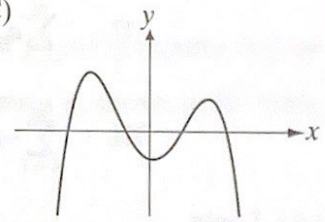
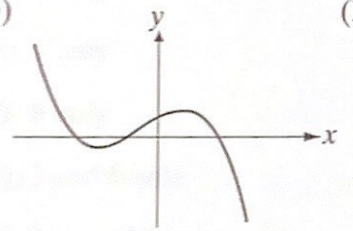
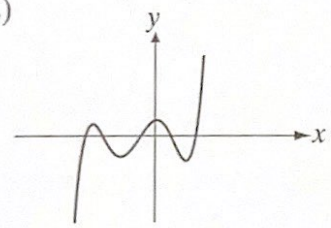


(D)



(E)



Answer

32. If $f(3) = 7$ and $f'(x) = \dfrac{\sin(1 + x^2)}{x^3 - 2x}$, then $f(5)$ is approximately

(A) -16.006 (B) -9.006 (C) -0.008 (D) 6.992 (E) 7.008

Answer

33. If $\sum_{n=1}^{\infty} |a_n|$ converges, then which of the following is true?

I. $\sum_{n=1}^{\infty} a_n$ converges.

II. $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

III. $\sum_{n=1}^{\infty} -a_n$ converges.

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

Answer

34. The base of a solid is the region enclosed by the graph of $y = 3(x-2)^2$ and the coordinate axes. If every cross section perpendicular to the x-axis is a square, then the volume of the solid is

(A) 8.0 (B) 19.2 (C) 24.0 (D) 25.6 (E) 57.6

Answer

x	1	2	3	4	5	6	7
$f'(x)$	5	0	-3	0	-2	0	4

35. Let $g(x) = \int_1^x f(t)\,dt$. The table above gives selected values of f', the derivative of f, on the interval $1 \le x \le 7$. If f' is continuous and has only the three zeros as shown in the table, for which values of x does g have a point of inflection?

(A) 2 only

(B) 4 only

(C) 6 only

(D) 2 and 6 only

(E) 2, 4, and 6

Answer

36. Let f be a function whose seventh derivative is $f^{(7)}(x) = 10{,}000 \cos x$. If $x = 1$ is in the interval of convergence of the power series for this function, then the Taylor polynomial of degree six centered at $x = 0$ will approximate $f(1)$ with an error of not more than

(A) 1.98×10^{-4} (B) 3.21×10^{-2} (C) 0.248 (D) 1.072 (E) 1.984

Answer

37. If f is an antiderivative of $\dfrac{\tan^2 x}{x^2 + 1}$ such that $f(1) = \dfrac{1}{2}$, then $f(0) =$

(A) 0 (B) 0.155 (C) 0.345 (D) 0.845 (E) 1

Answer

38. Suppose that $f(x)$, $f'(x)$, and $f''(x)$ are continuous for all real numbers x, and that f has the following properties.

I. f is negative on $(-\infty,6)$ and positive on $(6,\infty)$.

II. f is increasing on $(-\infty,8)$ and decreasing on $(8,\infty)$.

III. f is concave down on $(-\infty,10)$ and concave up on $(10,\infty)$.

Of the following, which has the least numerical value?

(A) $f'(0)$ (B) $f'(6)$ (C) $f''(4)$ (D) $f''(10)$ (E) $f''(12)$

Answer

39. The current price of a compact car is \$14,500. The price of a compact car is changing at a rate of $120 + 180\sqrt{t}$ dollars per year. What will be the approximate price of a compact car five years from now?

(A) \$15,020 (B) \$15,300 (C) \$16,440 (D) \$18,120 (E) \$22,600

Answer

40. If $0 \le k \le \frac{\pi}{2}$ and the area of the region in the first quadrant under the graph of $y = 2x - \sin x$ from 0 to k is 0.1, then $k =$

(A) 0.444 (B) 0.623 (C) 0.883 (D) 1.062 (E) 1.571

Answer

x	$f'(x)$
0.998	0.980
0.999	0.995
1.000	1.000
1.001	0.995
1.002	0.980

41. The table above gives values of the derivative of a function f. Based on this information, it appears that in the interval covered by the table

(A) f is increasing and concave up everywhere.

(B) f is increasing and concave down everywhere.

(C) f has a point of inflection.

(D) f is decreasing and concave up everywhere.

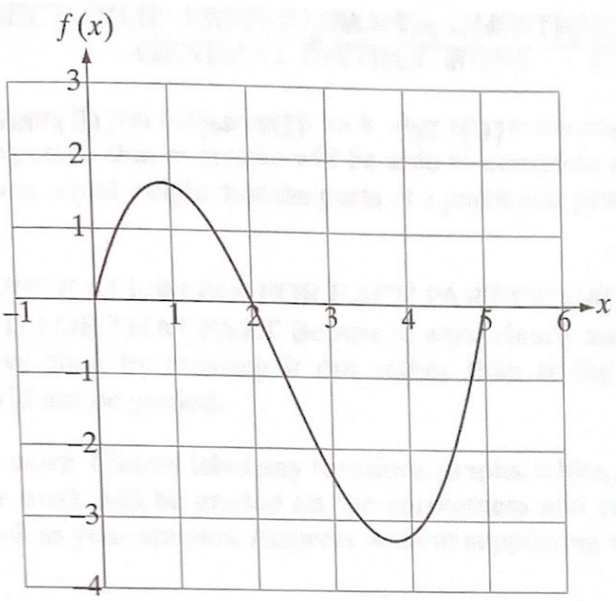
(E) f is decreasing and concave down everywhere.

Answer

42. The mass, $m(t)$, in grams, of a tumor t weeks after it begins growing is given by $m(t) = \frac{te^t}{800}$. What is the average rate of change, in grams per week, during the fifth week of growth?

(A) 0.273 (B) 0.341 (C) 0.619 (D) 0.655 (E) 1.113

Answer



43. The figure above shows the graph of a function $f(x)$ on the interval $[0,5]$. Which of the following definite integrals has the greatest value?

(A) $\int_0^1 f(x)\,dx$

(B) $\int_0^2 f(x)\,dx$

(C) $\int_0^3 f(x)\,dx$

(D) $\int_0^4 f(x)\,dx$

(E) $\int_0^5 f(x)\,dx$

Answer

44. If $g(x) = e^{2x}$, then $\lim_{h \to 0} \frac{g(1+h) - g(1-h)}{h}$ is

(A) 0 (B) e^2 (C) $2e^2$ (D) $4e^2$ (E) nonexistent

Answer

45. The closed interval $[0, \pi]$ is partitioned into n equal subdivisions each of length $\Delta x = \frac{\pi}{n}$ by the numbers $x_0, x_1, x_2 \ldots, x_{n-1}, x_n$, with $0 = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = \pi$.

The $\lim_{n \to \infty} \sum_{i=1}^{n} x_i \cos(x_i)\, \Delta x$ is

(A) –2 (B) –1 (C) 1 (D) 2 (E) π

Answer