TRUE-FALSE QUIZ

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

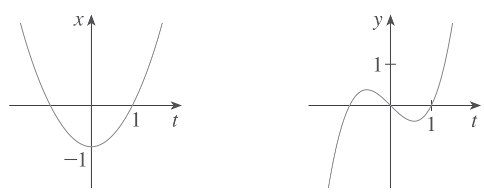
1. If the parametric curve $x = f(t)$, $y = g(t)$ satisfies $g'(1) = 0$, then it has a horizontal tangent when $t = 1$.
2. If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $d^2y/dx^2 = (d^2y/dt^2)/(d^2x/dt^2)$.
3. The length of the curve $x = f(t)$, $y = g(t)$, $a \leqslant t \leqslant b$, is $\int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt$.
4. If a point is represented by (x, y) in Cartesian coordinates (where $x \neq 0$) and (r, θ) in polar coordinates, then $\theta = \tan^{-1}(y/x)$.
5. The polar curves $r = 1 - \sin 2\theta$ and $r = \sin 2\theta - 1$ have the same graph.
6. The equations $r = 2$, $x^2 + y^2 = 4$, and $x = 2 \sin 3t$, $y = 2 \cos 3t$ $(0 \leqslant t \leqslant 2\pi)$ all have the same graph.
7. The parametric equations $x = t^2$, $y = t^4$ have the same graph as $x = t^3$, $y = t^6$.
8. The graph of $y^2 = 2y + 3x$ is a parabola.
9. A tangent line to a parabola intersects the parabola only once.
10. A hyperbola never intersects its directrix.

EXERCISES

1–4 Sketch the parametric curve and eliminate the parameter to find the Cartesian equation of the curve.

1. $x = t^2 + 4t, \quad y = 2 - t, \quad -4 \leqslant t \leqslant 1$
2. $x = 1 + e^{2t}, \quad y = e^t$
3. $x = \tan\theta, \quad y = \cot\theta$
4. $x = 2\cos\theta, \quad y = 1 + \sin\theta$

5. Write three different sets of parametric equations for the curve $y = \sqrt{x}$.
6. Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.



7–14 Sketch the polar curve.

7. $r = 1 - \cos\theta$
8. $r = \sin 4\theta$
9. $r = 1 + \cos 2\theta$
10. $r = 3 + \cos 3\theta$
11. $r^2 = \sec 2\theta$
12. $r = 2\cos(\theta/2)$
13. $r = \dfrac{1}{1 + \cos\theta}$
14. $r = \dfrac{8}{4 + 3\sin\theta}$

15–16 Find a polar equation for the curve represented by the given Cartesian equation.

15. $x + y = 2$
16. $x^2 + y^2 = 2$

17. The curve with polar equation $r = (\sin\theta)/\theta$ is called a **cochleoid**. Use a graph of r as a function of θ in Cartesian coordinates to sketch the cochleoid by hand. Then graph it with a machine to check your sketch.
18. Graph the ellipse $r = 2/(4 - 3\cos\theta)$ and its directrix. Also graph the ellipse obtained by rotation about the origin through an angle $2\pi/3$.

19–22 Find the slope of the tangent line to the given curve at the point corresponding to the specified value of the parameter.

19. $x = \ln t, \; y = 1 + t^2; \quad t = 1$
20. $x = t^3 + 6t + 1, \quad y = 2t - t^2; \quad t = -1$
21. $r = e^{-\theta}; \quad \theta = \pi$
22. $r = 3 + \cos 3\theta; \quad \theta = \pi/2$

23–24 Find dy/dx and d^2y/dx^2.

23. $x = t\cos t, \quad y = t\sin t$
24. $x = 1 + t^2, \quad y = t - t^3$

25. Use a graph to estimate the coordinates of the lowest point on the curve $x = t^3 - 3t$, $y = t^2 + t + 1$. Then use calculus to find the exact coordinates.

26. Find the area enclosed by the loop of the curve in Exercise 25.

27. At what points does the curve

$$x = 2a \cos t - a \cos 2t \qquad y = 2a \sin t - a \sin 2t$$

have vertical or horizontal tangents? Use this information to help sketch the curve.

28. Find the area enclosed by the curve in Exercise 27.

29. Find the area enclosed by the curve $r^2 = 9 \cos 5\theta$.

30. Find the area enclosed by the inner loop of the curve $r = 1 - 3 \sin \theta$.

31. Find the points of intersection of the curves $r = 2$ and $r = 4 \cos \theta$.

32. Find the points of intersection of the curves $r = \cot \theta$ and $r = 2 \cos \theta$.

33. Find the area of the region that lies inside both of the circles $r = 2 \sin \theta$ and $r = \sin \theta + \cos \theta$.

34. Find the area of the region that lies inside the curve $r = 2 + \cos 2\theta$ but outside the curve $r = 2 + \sin \theta$.

35–38 Find the length of the curve.

35. $x = 3t^2, \quad y = 2t^3, \quad 0 \leqslant t \leqslant 2$

36. $x = 2 + 3t, \quad y = \cosh 3t, \quad 0 \leqslant t \leqslant 1$

37. $r = 1/\theta, \quad \pi \leqslant \theta \leqslant 2\pi$

38. $r = \sin^3(\theta/3), \quad 0 \leqslant \theta \leqslant \pi$

39–40 Find the area of the surface obtained by rotating the given curve about the x-axis.

39. $x = 4\sqrt{t}, \quad y = \dfrac{t^3}{3} + \dfrac{1}{2t^2}, \quad 1 \leqslant t \leqslant 4$

40. $x = 2 + 3t, \quad y = \cosh 3t, \quad 0 \leqslant t \leqslant 1$

41. The curves defined by the parametric equations

$$x = \frac{t^2 - c}{t^2 + 1} \qquad y = \frac{t(t^2 - c)}{t^2 + 1}$$

are called **strophoids** (from a Greek word meaning "to turn or twist"). Investigate how these curves vary as c varies.

42. A family of curves has polar equations $r^a = |\sin 2\theta|$ where a is a positive number. Investigate how the curves change as a changes.

43–46 Find the foci and vertices and sketch the graph.

43. $\dfrac{x^2}{9} + \dfrac{y^2}{8} = 1$

44. $4x^2 - y^2 = 16$

45. $6y^2 + x - 36y + 55 = 0$

46. $25x^2 + 4y^2 + 50x - 16y = 59$

47. Find an equation of the parabola with focus $(0, 6)$ and directrix $y = 2$.

48. Find an equation of the hyperbola with foci $(0, \pm 5)$ and vertices $(0, \pm 2)$.

49. Find an equation of the hyperbola with foci $(\pm 3, 0)$ and asymptotes $2y = \pm x$.

50. Find an equation of the ellipse with foci $(3, \pm 2)$ and major axis with length 8.

51. Find an equation for the ellipse that shares a vertex and a focus with the parabola $x^2 + y = 100$ and that has its other focus at the origin.

52. Show that if m is any real number, then there are exactly two lines of slope m that are tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ and their equations are $y = mx \pm \sqrt{a^2m^2 + b^2}$.

53. Find a polar equation for the ellipse with focus at the origin, eccentricity $\frac{1}{3}$, and directrix with equation $r = 4 \sec \theta$.

54. Show that the angles between the polar axis and the asymptotes of the hyperbola $r = ed/(1 - e \cos \theta)$, $e > 1$, are given by $\cos^{-1}(\pm 1/e)$.

55. In the figure the circle of radius a is stationary, and for every θ, the point P is the midpoint of the segment QR. The curve traced out by P for $0 < \theta < \pi$ is called the **longbow curve**. Find parametric equations for this curve.